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ADAPTIVE TELEMETRY SYSTEMS
FINAL REPORT ON RESEARCH PERFORMED UNDER
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by

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This report summarizes the work performed under NASA Grant NGR 15-005-087 from May 1, 1968 to February 1, 1971.

Title: Exact Determination of Probability of Bit Error for Perfect Single Error Correcting Codes
Author(s): P. A. Wintz and G. G. Apple
Where Appeared: Proc. of the Third Hawaii International Conference on System Sciences, pp. 922-924, January 1970.

ABSTRACT

The binary perfect single error correcting codes were the first error correcting codes of any significance. These codes are very easy to implement and much is known about their structure. In this paper we give a simple method for calculating the exact probability of bit error after decoding for these codes.

Let G denote the code itself and G^i denote the subcode of G having a zero in the ith position. Then the probability of bit error (after decoding) in the ith position is the probability that an element of G^i is transmitted and an element of $G-G^i$ is received. Since G is a group code this can be shown to be equal to the probability that 0 (the all zero codeword) is transmitted and an element of $G-G^i$ is received. The probability that 0 is transmitted and $g \in G$ is received can be shown to

be $Q_j = q^n(p/q)^j[1 + j \frac{q}{p} + (n-j) \frac{p}{q}]$ where j is the Hamming weight of g, p is probability of a channel bit error and q = 1-p. Letting

$A^i(z) = \sum_{j=0}^{n} A_j^i z^j$ denote the weight enumerator of $G-G^i$ ($A^i(z)$ turns out to be independent of i) we can calculate the probability of error in the ith bit as

$$P_e(i) = \sum_{j=0}^{n} A_j^i Q_j$$

Thus the probelm reduces to that of obtaining $A^i(z)$. We show that $A^i(z)$ is very easy to evaluate and is given by

$$A^i(z) = \frac{z}{n+1}\left[(1+z)^{n-1} + (1-z^2)^{\frac{n-3}{2}}(1-z)(nz-1)\right].$$

Title: Effect of Noncoherent FSK Versus PSK on Waveform Error in PCM Telemetry
Author(s): P. A. Wintz and A. J. Kurtenbach
Where Appeared: IEEE Trans. on Aerospace and Electronics, Vol. AES-6, No. 3, pp. 324-333, May 1970.

ABSTRACT

In a recent paper [1] the authors presented an optimization strategy for the transmission of analog data over a PCM telemetry link. Constraints imposed included real-time operation and both average and peak power limitations on the transmitter. The selected strategy used Karhunen-Loeve (KL) sampling, unequal bit assignmnets, optimum uniform quantizing, the natural code, and PSK reception.

In this paper, the performance of the above system will be compared to the same system using noncoherent FSK reception instead of PSK. The performance criterion used for the comparison is the mean integral squared error criterion.

Title: Fast Multipliers
Author(s): P. A. Wintz and A. Habibi
Where Appeared: IEEE Trans. on Computers, Vol. C-19, No. 2, pp. 153-157 (SN), February 1970

ABSTRACT

A number of schemes for implementing a fast multiplier are presented and compared on the basis of speed, complexity, and cost. A parallel multiplier designed using the carry-save scheme and constructed from 74 series integrated circuits is described. This multiplier multiplies 10 bit x 12 bit binary numbers with a worst case time of 520 nano-seconds. The cost of the integrated circuits was less than $500.

Title: Calculation of Fourier Transforms on Finite Abelian Groups
Author(s): P. A. Wintz and G. G. Apple
Where Appeared: IEEE Trans. on Information Theory, Vol. IT-16, No. 2, pp. 233-234, March 1970.

ABSTRACT

A recent paper by Crimmins, et. al., deals with minimization of mean-square-error for group codes by the use of Fourier transforms on groups.

Title: BCH Code Performance for Sampled Data
Authro(s): P. A. Wintz and G. G. Apple
Where Appeared: 1970 IEEE International Communications Conference, June 1970

ABSTRACT

The problem of evaluating the mean-square-error performance of sampled data systems employing error correcting codes is considered. Performance curves are given for most of the primitive binary BCH codes of lengths through 127. It is established that for uniformly distributed data samples the mean-square-error due to a noisy channel is determined solely by the probability of bit error after decoding. This is true even though redundancy in the form of parity check bits is used in encoding the samples. A method is given for determining the bit error probability for the BCH codes over GF($\hat{q}$) when a $\hat{q}$-ary equidistant signal set is used.

Title: Analysis of Differential Systems for PCM Transmission
Author(s): P. A. Wintz and B.R.N. Murthy
Where Appeared: Conference Record, 1970 IEEE International Conference on Communications, pp. 45-55, June 1970

ABSTRACT

In this paper we consider differential systems that employ PCM (Pulse Code Modulation) to transmit the difference between actual and predicted information samples. Three system configurations are investigated. In the first approach, the incoming sample is predicted based on all the proceding samples before quantization. In the second method, prediction is made on the basis of quantized data. This second method, more commonly called differential PCM (DPCM), uses a feedback path around the quantizer. In the third system, prediction of the incoming samples is made on the basis of the past received data. We assume that a noiseless feedback channel from the receiver to the transmitter is available. The three differential systems are optimized under a mean squared error criterion.

We first obtain in each case the structure of the optimum predictor for the case of stationary data. Although the optimum predictor in general is time varying, at low error rates (less than 10-3) it can be approximated by a fixed form. Assuming a noisy channel we then obtain for each system a closed form expression for the minimum mean squared error. The transient effects during the system start-up are also examined.

Specific results are presented for the case of first order Gaussian Markov data.

Title: Solution to a Limited Occupancy Problem
Author(s): P. A. Wintz and L. C. Wilkins
Where Appeared: (submitted for publication)

ABSTRACT

A formula for the number of ways of distributing r indistinguishable balls among n cells such that no cell contains more than L balls is derived.

Title: Performance of PCM Systems Employing Error Correcting Codes
Author(s): P. A. Wintz and G. G. Apple, Jr.
Where Appeared: TR-EE 70-13, School of Electrical Engineering, Purdue University, Lafayette, Indiana, August 1970.

ABSTRACT

Transmission of analog data by PCM systems employing error correcting codes is considered. The analog samples are quantized, encoded with an error correcting code, transmitted over a binary symmetric channel, decoded, and reconstructed. Analytical results and performance curves are given for several classes of codes among which are simplex codes, Hamming codes, BCH codes, threshold decodable convolutional codes, threshold decodable block codes and probabilistically decoded convolutional codes. The use of these codes for unequal error protection is also considered. The distortion measure used throughout is mean square error.

It is shown that for uniformly distributed data the mean square error due to the channel depends exclusively on the probability of bit error (after decoding). For this reason a considerable emphasis is placed on evaluating the probability of bit error for the given classes of codes. Performance curves are given for the simplex codes. An entirely new technique is given for determining the bit error probability for binary Hamming codes and for Hamming codes over GF (q). A method is derived for determining the probability of bit error for BCH codes over GF (q) for which the weight enumerators are known. In both of these cases performance curves are given for a large number of useful codes. A method based on Massey's work is given for determining the bit error probability of threshold decoded convolutional codes. Exact curves are given for the self-orthogonal codes and "first-error" probbability curves are given for some other codes. The same technique is shown to apply to threshold decoded block codes and performance curves are given for a very good Euclidean geometry code. Curves are given for codes decoded with the Viterbi algorithm. Some simulation results for sequentially decoded convolutional codes are also given for purposes of comparison. It is shown that for real time (constant information rate) transmission the higher rate codes in general perform better than the lower rate codes.

The total mean square error of the coded system is determined. An expression for the quantization error as a function of the number of quantization bits is determined. A concise expression is determined for the mean square error due to the channel. It is shown that the total mean square error is given by the sum of the quantization error and the channel error. Mean square error curves are given for samples encoded with Hamming codes and BCH codes with the number of quantization bits as a parameter. Again the higher rate codes tend to outperform the lower rate codes.

An unequal error protection scheme known as bit-plane encoding is evaluated for several types of codes. A method is given for determining the actual code rate and an expression is derived which enumerates the total number of cases which need to be examined. It is shown that by proper ordering the total number of cases is reduced to a workable number. The general technique is to provide more redundancy for the higher order bits than for the lower order bits. Thus, more of the error protection is allotted to the positions where a bit error causes the greatest contribution to the mean square error. The use of Hamming codes in the highest order positions is examined. Performance curves are given and the optimum distribution of coding and transmitted bits is given. It is shown that the repetition code can provide useful gain when used as a UEP code even though repetition codes are uniformly bad when used for complete coding. Multiple error correcting BCH codes are also considered. For instance a 3-error correcting code might be used on the highest order bits. Then double and single error correcting codes could be used on successively lower order bits. Optimum combinations of such codes are determined and are tabulated for different signal-to-noise ratios. Rate one-half self-orthogonal convolutional codes are also considered. However, in this case it appears to be best to use the most powerful code available on all higher order bit positions rather than a more general distribution of error protection.

Title: Subset Codes for Protection From Synchronization Loss and Substitution Errors
Author(s): P. A. Wintz and G. Robert Redinbo
Where Appeared: Unpublished

ABSTRACT

Block codes are developed which have the capability of protecting against the simultaneous occurrence of substitution errors and the loss of word synchronization by an integral number of positions. Protection refers to either the detection, the detection and classification, or the correction of both types of errors. The results are given as the maximum number of substitution errors and the maximum number of digits of misframing for which the block code is guaranteed to afford protection. The misframing is permitted in either direction and the range which is protected need not be symmetrical about the true synchoronization position.

The block code is derived from a cyclic code by removing certain of its vectors. The guiding principle is to delete those vectors which are most sensitive to loss of synchronization. The remaining subset of vectors is the block code.

The rate of the block code is less than that of original cyclic code, and its substitution error-detecting or error-correcting efficiency is also reduced whenever both types of errors occur together. There is a trade-off between the performance measures of rate, error-protecting efficiency and word synchronization protection. However if synchronization is maintained, the block code's error performance can exceed that of the parent code.

Title: A Survey of Data Compression Techniques
Author: P. A. Wintz
Where Appeared: 1970 Southwestern IEEE Conference Record, pp. 29-32, Dallas, Texas, April 1970.

ABSTRACT

A survey of data compression techniques is presented. The problem of coding 2-dimensional data (images) is given special consideration.

Title: Optimum Adaptive Reception for Binary Sequences
Author: P. A. Wintz
Where Appeared: IEEE Trans. on Aerospace and Electronic Systems, Vol. AES-6, No. 3, pp. 334-339, May 1970.

ABSTRACT

The a posteriori probability density function $p(\theta|X_1, X_2, \ldots, X_K)$, where the X_i, $i=1,2, \ldots,K$, represent K vector-valued observations statistically related to the random vector θ, appears in many applications of the methods of statistical inference to problems in pattern recognition and statistical communication theory. In this paper, it is shown that for equally likely binary sequences (M-2) of anticorrelated patterns for signals observed in additive Gaussian noise, a device that computes $p(\theta|X_1, X_2, \ldots,X_K)$ can be synthesized from a correlator, a simple instantaneous nonlinearity, and a multiplier. These results are used to derive some equally simple structures for various optimum nonsupervised estimators, pattern recognition machines, and signal detectors.

Title: Coding for Quantized Samples
Author(s): P. A. Wintz and G. G. Apple, Jr.
Where Appeared: Proc. of the UMR-Marvin J. Kelly Communications Conference Univ. of Mo., Rolla, Mo., October 1970.

ABSTRACT

Source encoding and channel encoding are usually considered separately. In this paper we examine the combined effects of the two types of encoding when uniform quantization and binary group codes are used. Unequal error protection by means of bit-plane encoding using BCH codes is considered. A technique is given for selecting the best combination of codes (in the mean square error sense) from a particular class of codes.

Title: An Experimental PCM System with Digital Filtering for Data Compression
Author(s): P. A. Wintz and G. G. Apple, Jr.
Where Appeared (Submitted for Publication)

ABSTRACT

An experimental PCM system employing digital filters for data compression, unequal bit assignments, and error control coding was constructed from integrated circuits. The digital filter is capable of implementing Karhunen-Loeve, Hadamard, and other transformations. We first give a general description of the entire system. Then we discuss the individual components in more detail.

Title: On the Statistics of Correlator Outputs
Author: P. A. Wintz
Where Appeared: (Abstract published in IEEE Trans. on Information Theory, Vol. IT-12, No. 2, pp. 279, April 1966).

ABSTRACT

The characteristic function, probability density function, and probability distribution function are derived for the random variable z where

$$z = \sum_{j=1}^{2N} (s_j + n_j)(r_j + m_j).$$

The s_j and r_j are arbitrary numbers and the n_j and m_j are gaussian random variables. The probability Prob$[z < 0]$ is plotted for selected values of the pertinent parameters.

size, the total number of points on and within a contour, and the number of points represented by a contour.

The concept of interior and exterior contours leads to very interesting results concerning the distribution of these types of contours in a typical picture. Several pictures were made with the short exterior contours turned white in order to clearly mark their positions. Similar pictures were created marking the location of short interior contours. The results of these studies indicate that the exterior contours typically occur in areas which are the most important subjectively. This is clearly demonstrated by a sequence of pictures in which short interior contours were deleted with little or no perceptible degradation. Corresponding pictures in which exterior contour were removed yielded appreciable degradation.

The improved overall bit compression ratio for several pictures with interior deletions is presented with the accompanying photographs.

Title:	Studies on Data Compression: Part II Error Analysis of Run-Length Codes
Author(s):	P. A. Wintz and L. C. Wilkins
Where Appeared:	Presented at the Symposium on Picture Coding, North Carolina State University, Raleigh, N.C., September 10, 1970.

ABSTRACT

Most redundancy removal algorithms employ some sort of run length code. Blocks of timing words are coded with synchronization words inserted between blocks. The probability of incorrectly reconstructing a sample because of a channel error in the timing data is a monotonically non-decreasing function of time since the last synchronization word.

In this we compute the "probability that the accumulated magnitude of timing errors equal zero" as a function of time since the last synchronization word for a zero order predictor. The analysis is based in part on some results on Semi-Markov processes due to Pyke. The analysis is valid for a data compression system employing a floating aperture zero-order redundancy remover with a limit ℓ on the maximum run length. (This constraint results from practical coding considerations and greatly complicates the analysis). The data source is modeled by a first order Morkov chain with any valid transition matrix, and the digital channel is modeled by a channel transition matrix.

This general analysis is then applied to a particular data compression system, in which both the data source and the channel are characterized by transition matrices having constant main diagonals. Results are obtained for several values of ρ (source correlation) and Δ (channel error). A family of five curves corresponding to run length constraints of 4,8,16,32, and 64 is plotted for each ρ,Δ pair. These curves enable the system designer to select the synchronization block length so as to maximize the compression ratio while maintaining the required data quality.

In this paper we show how to represent the groups in a form suitable for machine calculation. We also propose an efficient method for calculating the fourier transform of a group and show its relationship to the Fast Fourier transform. For groups of characteristic two, the calculation requires only $N \log_2 N$ additive operations where N is the order of the group.

Title: A Feedback Communication System for Deep Space Applications
Author(s): P. A. Wintz and J. L. Katz
Where Appeared: NASA Coded Communications Conference, NASA/JPL, Pasadena, California, February 1970.

ABSTRACT

An adaptive telemetry system based on a feedback coding scheme for very deep space application (e.g., The Grand Tour) is described. The downlink uses convolutional encoding sequential decoding. The sequential decoder can use a maximum of C_{max} computations to decode a frame. Data frames decoded by the decoder are accepted as correct. (For $E_b/N_o = 2$ db with a 1/2 rate code, 32 bit constraint length, the undetected bit error rate is approximately 1×10^{-4} but the frame deletion rate is approximately .2). If a frame is not decoded an attempt is made to decode the data sequence in the reverse direction. If this fails a retransmission is requested via the uplink (feedback path).

System performance depends critically on the partitioning of energy between the intital transmission and retransmission. The total available energy per bit, Eb, must be partitioned such that ρE_b is used for the initial transmission and $(1-\rho)E_b$ reserved for retransmissions.

It is shown that for $E_b/N_o < 2.1$ db the optimum partition is $\rho=1$. For $E_b/N_o > 2.1$ db. $\rho_{opt} < 1$ indicating that the feedback coding scheme improves performance. Curves showing frame deletion rate via E_b/N_o and ρ for three different schemes for processing retransmitted frames are presented: 1) the transmitted data alone is processed; 2) a weighted average of the initial and retransmitted signal, is processed; 3) a 1/4 rate code is formed by combining the two 1/2 rate codes and decoded.

Other design curves are also presented. These include showing the improvement in the operation of the decoder using the forward-reverse decoding scheme as well as the improvement in the Pareto distribution exponent due to feedback.

Title: Codes for Protection from Synchronization Loss and Additive Errors

Author(s): P. A. Wintz and G. R. Redinbo

Where Appeared: TR-EE 69-43 School of Electrical Engineering, Purdue University, Lafayette, Indiana, November 1969.

ABSTRACT

Cyclic codes are practical and efficient codes which protect against the effects of additive errors. However their effectiveness, like that of block codes with symbols from a general finite field are modified so that they are also capable of protecting against misframing at the decoder. These codes are modified by altering their distance structure. There are a number of techniques which can be employed. Each method affects different aspects of the code's performance; therefore a complete and comprehensive coverage of all techniques is given.

Results for each modification approach are given for three types of protection from the simultaneous occurrence of additive errors and synchronization errors. The first type is the detection of some kind of error, the second is the detection and the classification of the nature of the error, and the third is the correction of both kinds of errors. Furthermore for each approach results are presented for the cases of symmetrical and unsymmetrical ranges of synchronization errors. The proofs of all results indicate the general strategy for decoding the modified code.

A coset of the original code allocates part of its error-protecting capabilities to synchronization. Results are given for the general class of cyclic codes. Stronger conclusions are possible when the special case of Reed-Solomon codes is considered. In this case protection from slips of half the code's length in either direction are permitted.

A subset code is derived from a code by removing certain of its vectors so as to produce a code with fewer members which are less sensitive to misframing. Two approaches to subset codes are demonstrated. One is a coset code of an expurgated code while the other is a code with a fixed pattern imbedded in the information digits.

Changing the length of a code when combined with other techniques is another modification approach. The work here improves on the few known results and introduces many new ones so as to complete and consolidate all aspects of this type of approach. Results concerning shortened codes are developed, subset codes are extended to yield another modification approach, and coset codes are lengthened to produce a new scheme.

Two approaches for achieving wide-range slip protection are presented. One uses interleaving while the other combines interleaving with concatenation. With either technique slip protection ranges of half the code's length are possible. The interleaving technique may be coupled with any other approach giving the net effect of greatly expanding the slip protection range of that approach. Combining concatenation and interleaving accomplishes the same result without increasing the complexity of the encoder and decoder to the extent to which they would be if only interleaving were used. It is shown that for wide-range slip protection the error-protecting performance of either approach is superior to any other known approach.

Title:	Studies on Data Compression: Part I. Picture Coding by Contours
Author(s)	P. A. Wintz and L. C. Wilkins
Where Appeared:	Presented at the Symposium on Picture Coding, North Carolina State University, Raleigh, N.C., September 10, 1970.

ABSTRACT

A method is presented for reducing the number of bits required to describe a quantized 2-dimensional data array by making use of its area properties. The technique is also useful for gathering statistics on area-related parameters and for data processing such as image enhancement. The operation of the technique is demonstrated on several photographs which have been appropriately sampled and quantized.

The algorithm locates and traces all contours - outer boundaries of areas having the same grey level - of any quantized 2-dimensional data array. For each contour, the system outputs the value and location of the initial point and the direction of travel from element to element as the contour is traced in the clockwise direction.

A reconstruction algorithm is described which perfectly reconstructs the original data array from the output described above.

Several modifications of the standard algorithm are described. Every data array is composed primarily of contours which fit together like pieces of a jigsaw puzzle. There are, of course, some other contours which are completely embedded within a single contour. The former contours are called exterior, the latter: interior. This classification is the basis for two modifications of the original algorithm. The modifications allow us to delete either directional information (shape), or position information of many contours and yet reconstruct the original array perfectly. Statistical coding techniques are applied to the contour tracer output, and performance comparisons are made for the various codes.

The Contour Tracing Algorithm is quite useful in gathering statistics on area-related parameters. Techniques are described for determining area

Title: Comment on Optimization of PCM Systems which use Natural Binary Codes
Author(s): P. A. Wintz and A. J. Kurtenbach
Where Appeared: Proc. of the IEEE, Vol. 57, No. 4, pp. 711-712, April 1969 (Letter)

In the above letter,[1] Donaldson discussed the optimization of PCM systems which use natural binary codes. Some of the author's statements are subject to further qualification and several of his approximations are not consistant with other published results. The points to which we take exception are elaborated below.

The author quotes a result of Huang[2] that natural binary codes are optimum for $N \leq 3$ (N is the number of bits per sample) and $p \leq \frac{1}{4}$ (p is the probability of bit error on the BSC) in the sense that the MSE (mean square error) due to channel bit errors is less than for any other fixed length nonredundant binary code. First, this result is valid only for input data uniformly distributed over the integers 0 to $2^N - 1$. Indeed, it has beendemonstrated[3] that for the case of Gaussian data quantized using the optimum uniform quantizer[4] and transmitted over the BSC, the binary reflected Gray code yields smaller MSE than the natural code for high signal-to-noise ratios in the channel. Second, the statement is valid for all N (as opposed to $N \leq 3$) and for all perfect codes (redundant as well as nonredundant) as shown by Crimmins, et al.[7]

Equation (1) of Donaldson gives an expression for the total MSE between the input and output waveforms of a PCM system. The three contributing terms are due to the input filter and sampler, the quantizer and the channel. A comment follows to the effect that in a practical situation the error due to the quantizer is not strongly dependent upon the sampling rate. We point out that Totty and Clark[8] have shown that the quantization error is linearly dependent upon the sampling rate. Their development is quite general and includes the system described by Donaldson as a special case. We also point out that the general expression for the total system MSE contains a fourth quantizer error-cross-channel error term that is zero if and only if Max's uniform quantizer is employed.

Finally, the PCM system discussed by Donaldson has been optimized for more general sampling and reconstruction strategies and its performance evaluated for both the MSE and rate distortion criteria.[9] In particular filtering were compared to time sampling for stationary Gaussian inputs. Digital data conditioning the time samples before transmission yields considerable improvement in performance.

Title: Log-Linear Companding - A Digital Companding Technique
Author(s): P. A. Wintz and G. G. Apple
Where Appeared: Proc. of the IEEE, (Letter), Vol. 57, No. 10, pp. 1776-1777, October 1969

ABSTRACT

A new method of near-logarithmic companding is proposed. The method consists of first uniformly quantizing each sample and then processing the resulting binary number digitally. This is in contrast to the usual scheme of first compressing the input and then uniformly quantizing it. The method presented here is extremely simple to implement and requires no arithmetic operations. The inverse transformation is equally simple to implement.

Title: Optimum Transmission of Digital Data by Pulse Code Modulation
Author(s): P. A. Wintz and B.R.N. Murthy
Where Appeared: Proc. of the 1969 National Telemetry Conference, April 1969

ABSTRACT

The problem of optimal transmission of digital data by pulse code modulation is considered in this paper for minimum m.s.e. criterion. A simple, yet general, method of optimum uniform quantization for noisy channels is first presented and applied to quantize gaussian and uniformly distributed data. Then, two general methods of deciding optimum word lengths for the data while transmitting at a constant bit rate over a finite time are presented, using the techniques of dynamic programming and classical optimization. Specific results are obtained for data from a gaussian process with an autocorrelation function of $e^{-|\tau|}$.

Title: A Contour Tracing Algorithm for Data Compression for Two Dimensional Data
Author(s): P. A. Wintz and L. C. Wilkins
Where Appeared: Accepted for National Electronics Conference December 8,9,10, 1968.

ABSTRACT

An algorithm for locating and tracing all contours of any 2-dimensional data array is described. The algorithm consists of two subalgorithms - an IP algorithm for locating new initial points (new contours) and a T algorithm for tracing contours after they are located. The subalgorithms are used sequentially - the IP algorithm is used to locate the first IP (initial point on the first contour, and then the T algorithm is used to trace it; then the IP algorithm is used to locate the second IP (initial point on the second contour), and then the T algorithm is used to trace it; etc. The IP algorithm locates all contours; none are located twice. The T algorithm traces the outer boundary of the largest connected set of elements having the same value as the initial point; it always terminates back at the initial point. For each contour, the system outputs the value and location of the IP and the direction of travel to each element on the contour. All elements enclosed by the contour and having the same value as the contour are neglected. An algorithm for reconstructing the original data array, without distortion, from the system output is also described.

Title: Optimum Linear Transformations for Encoding 2-Dimensional Data
Author(s): P. A. Wintz and Ali Habibi
Where Appeared: Presented at the "Symposium on Picture Bandwidth Compression," MIT, Cambridge, Mass.; April 1969.

ABSTRACT

Optimum (mean square error) linear transformation for transforming 2-dimensional data arrays into 1-dimensional data arrays are developed. For continuous data the development is analogous to the Karhunen-Loeve expansion for 1-dimensional continuous data, and for discrete data the development is based on Hotelling's Method of principal components. The performances of both methods are evaluated and compared to the performances of various other source encoding strategies for the 2-dimensional gaussian process with autocorrelation function $R(x,x',y,y') = \exp(-\alpha|x-x'|-\beta|y-y'|)$. Performance comparisons are made on the basis of mean square error and rate distortion function.

Title: n-th Order Redundancy Reduction
Author(s): P. A. Wintz and L. B. Wilkins
Where Appeared: Proc. of the 1968 National Electronics Conference, Dec. 1968.

ABSTRACT

One method of effecting redundancy reduction on sampled data is to fit (within a specified tolerance) an nth order polynomial to the first i data points, another nth order polynomial to the next j data points, etc. Then the parameters of these polynomials and the numbers i,j,k,... are transmitted rather than the data points themselves. A general technique for fitting the curves described above in such a way as to absorb the largest possible number of data points into each curve is given. Two special cases of great practical significance are also presented.